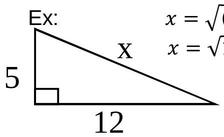
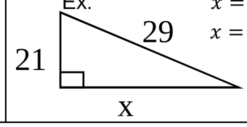
Math 306 Group _____ Memory Aid Teacher: Mrs. Botros

Chapter 1: Real numbers

- Pythagorean theorem, to find:
  - Hypotenuse: $c = \sqrt{(a^2 + b^2)}$
    Ex: $x = \sqrt{(5^2 + 12^2)}$, $x = \sqrt{169} = 13$

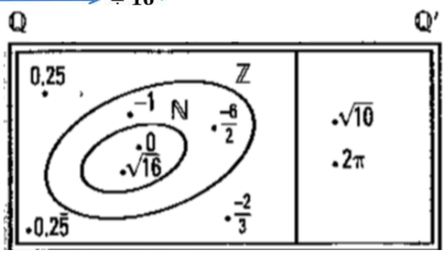


  - Leg: $a = \sqrt{(c^2 - b^2)}$
    Ex: $x = \sqrt{(29^2 - 21^2)}$, $x = \sqrt{400} = 20$



- Changing repeating decimal to fraction:
  Ex: $0.1\bar{6} \rightarrow 1.\bar{6} = 1\frac{6}{9} = 1\frac{2}{3} = \frac{5}{3} \rightarrow \frac{5}{30}$ $= \frac{1}{6}$
  (× 10 → ÷ 10)

- ∈ : element of
  ⊆ : subset of
  $N \subseteq Z \subseteq Q \subseteq R$



- Laws of exponents:
  $A^0 = 1$ $a^1 = a$

	Law	Example
1.	$a^m a^n = a^{m+n}$	$x^2x^3 = x^5$
2.	$\frac{a^m}{a^n} = a^{m-n}$ if m>n	$\frac{x^6}{x^2} = x^4$
	$= \frac{1}{a^{n-m}}$ if n>m	$\frac{x^2}{x^6} = \frac{1}{x^4}$
3.	$(a^m)^n = a^{mn}$	$(x^2)^3 = x^6$
4.	$(ab)^n = a^n b^n$	$(2x)^3 = 8x^3$
5.	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{x}\right)^3 = \frac{8}{x^3}$
6.	$a^{-n} = \frac{1}{a^n}$	$x^{-3} = \frac{1}{x^3}$
7.	$a^{1/n} = \sqrt[n]{a}$	$x^{1/3} = \sqrt[3]{x}$

- Scientific notation
  $a \times 10^n$
  $(1 \leq a < 10)$
  Ex:
  $(2.3 \times 10^{-3})(7.9 \times 10^5)$
  $= (2.3)(7.9) \times (10^{-3+5})$
  $= 18.17 \times 10^2$
  $= 1.817 \times 10 \times 10^2$
  $= 1.817 \times 10^3$

Chapter 2: Algebraic Expressions

- Definitions:
  1. Monomial: 1 term either variable, constant, or product
  2. Polynomial: more than one term separated by +/- ; all terms have variables with natural exponents
     (a) Binomial- 2 terms (b) trinomial- 3 terms
  3. Coefficient: is the number preceding the variable(s)
  4. Like terms: have identical variables with identical exponents
  5. Degree: (a) of a monomial: sum of exponents of all variables
     (b) of a polynomial: the degree of the term with highest degree
  6. Zero of a polynomial: is the value of the variable that makes the polynomial = 0
  7. Simplify: is to collect (add or subtract) all like terms to have fewer terms
  8. Evaluate: is to replace the variable with a given value & follow BEDMAS

- Adding/Subtracting Polynomials: coefficients only, exponents don't change

Ex 1: $(2x^2 + 5x + 8) + (x^2 - 4x + 5)$ $= 3x^2 + x + 13$	Ex 2: $(2x^2 + 5x + 8) - (x^2 - 4x + 5)$ $= x^2 + 9x + 3$

- Multiplying Polynomials:

Case 1 Distributive	Case 2 F O I L	
Ex: $3x(4x + 2)$ $= 12x^2 + 6x$	Ex: $(3x + 2)(4x - 5)$ F O I L $= 12x^2 - 15x + 8x - 10$ $= 12x^2 - 7x - 10$	Ex: $(3x + 2)^2$ $= (3x + 2)(3x + 2)$ $= 9x^2 + 6x + 6x + 4$ $= 9x^2 + 12x + 4$

- Dividing Polynomials: Divide each term by the monomial
  Ex: $\frac{20xy^5 - 15xy^2 + 30x^2y^4 + 5xy}{5xy}$
  $= 4y^4 - 3y + 6xy^3 + 1$

- Greatest Common Factor:
  1. Find the GCF for the coefficients and each variable (the variables with the smallest exponents)
  2. To get the second factor, divide the polynomial by your GCF
     Ex: $18x^4y + 12x^3y^2$
     $= (6x^3y)\left(\frac{18x^4y + 12x^3y^2}{6x^3y}\right)$
     $= (6x^3y)(3x + 2y)$

Chapter 3: Equations & Inequalities

	Equations	Inequalities/ interval solution	EFF
1	$x + 3 = 7$ $x = 4$	$x + 3 < 7$ $x < 4$	Interval:]-∞,4[4
2	$2x + 4 = 10$ $2x = 6$ $x = 3$	$2x + 4 \geq 10$ $2x \geq 6$ $x \geq 3$	Interval: [3,∞[3
3	$5x + 25 = -3x - 23$ $8x + 25 = -23$ $8x = -48$ $x = -6$	$5x + 25 > -3x - 23$ $8x + 25 > -23$ $8x > -48$ $x > -6$	Interval:]-6,∞[-6
4	$6(x - 2) = -4(2x + 1)$ $6x - 12 = -8x - 4$ $14x - 12 = -4$ $14x = 8$ $x = \frac{8}{14} = \frac{4}{7}$	$6(x - 2) \leq -4(2x + 1)$ $6x - 12 \leq -8x - 4$ $14x - 12 \leq -4$ $14x \leq 8$ $x \leq \frac{8}{14} = \frac{4}{7}$	Interval: $]-\infty, \frac{4}{7}]$ $\frac{4}{7}$
5	$\frac{3x+5}{4} = \frac{4x+10}{2} + 1$ $\frac{3x+5}{4} = \frac{2(4x+10)}{4} + \frac{4}{4}$ $3x + 5 = 8x + 20 + 4$ $-5x + 5 = 24$ $-5x = 19$ $x = -\frac{19}{5}$	$\frac{3x+5}{4} \leq \frac{4x+10}{2} + 1$ $\frac{3x+5}{4} \leq \frac{2(4x+10)}{4} + \frac{4}{4}$ $3x + 5 \leq 8x + 20 + 4$ $-5x + 5 \leq 24$ $-5x \leq 19$ ⇦★ $x \geq -\frac{19}{5}$	Interval: $[-\frac{19}{5}, \infty[$ $-\frac{19}{5}$ **Exception: when you divide by a negative, flip the sign.**

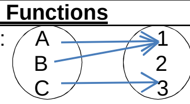
More examples:

$-5(2x+1) + 3(x-2) = 2(4x-1) - 3(2x-3)$ $-10x - 5 + 3x - 6 = 8x - 2 - 6x + 9$ $-7x - 11 = 2x + 7$ $-9x - 11 = 7$ $-9x = 18$ $x = -2$	$5(2x+1) + 3(x-2) \geq 2(4x-1) - 3(2x-3)$ $-10x - 5 + 3x - 6 \geq 8x - 2 - 6x + 9$ $-7x - 11 \geq 2x + 7$ $-9x - 11 \geq 7$ $-9x \geq 18$ $x \leq -2$

$80 < 4x + 20 < 100$
$60 < 4x < 80$
$15 < x < 20$
Interval solutions is:]15,20[
15 20

Chapter 4: Functions

A relation:



Input	Output
Source	Target
Antecedent	Image
Independent	Dependant
x-value	y-value
Domain	Range

It is a **function** if there is at most one y-value for every x-value.

VLT: (Vertical line test) in a function the VLT touches the graph at most once.

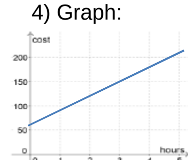
Notation: f(x) = y

Modes of representation:
1) Written description EX: Repairman charges $30 per hour plus $60 for travel.
2) Rule/Equation: f(x) = 30x + 60; indep. var.(x) time ; dep var. (y) cost
3) Table of values: (TOV) Δx = 1 1 1

X(hours)	0	1	2	3
Y(cost)	60	90	120	150

Δy = 30 30 30

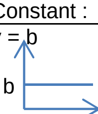
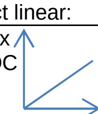
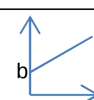
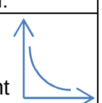
4) Graph:



Rate of change:

ROC $= \frac{rise}{run}$ (given graph)
$= \frac{\Delta y}{\Delta x}$ (given TOV) EX: $\frac{\Delta y}{\Delta x} = \frac{30}{1} = 30$
$= \frac{y_2 - y_1}{x_2 - x_1}$ (given 2 points) EX: A(3,6) and B(5,-2) ROC $= \frac{-2-6}{5-3} = \frac{-8}{2} = -4$

Types of functions:

Constant :	Direct linear:	Partial linear:	Rational:
y = b b 	y = ax a:ROC b= 0 	y = ax + b a:ROC b: y-intercept or initial value 	$y = \frac{C}{x}$ x·y = C Constant product 

Finding the rule/equation of the line: y = ax + b

EX:

X	Y
5	280
8	406

Step 1: find a (ROC)	Step 2: find b (initial value)
$a = \frac{406-280}{8-5} = \frac{126}{3} = 42\ \$/hr$	$b = y_1 - ax_1$ = 280 – (42)(5) = 70$

Therefore: f(x)= y= 42x + 70

Solving a system of equations:

$\begin{cases} y_1 = 2x + 5 \\ y_2 = x + 8 \end{cases}$	$y_1 = y_2$ $2x + 5 = x + 8$ $x = 3$	$y_1 = 2(3) + 5$ $= 11$	$y_2 = (3) + 8$ $= 11$

Therefore the solution is (3,11) : for x = 3 the y = 11 for both equations

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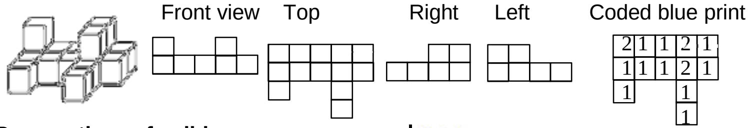
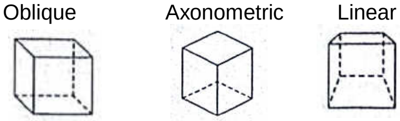
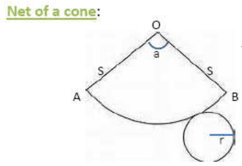
Chapter 5: Solids



Perspectives of solids:





$\frac{a}{360°} = \frac{mAB}{2\pi s}$

$\frac{a}{360°} = \frac{r}{s}$

A prism: has only 2 parallel and congruent bases, its named after the base, its height is the distance between the bases; can be generated by translating the base

A cylinder: has only 2 parallel and congruent disks, its height is the distance between the bases, can be generated by translating a disk, or rotating a rectangle along one of its edges.

A Pyramid: has 1 base and an apex, its named after the base, it has a height and a slant height $s^2 = h^2 + (b/2)^2$ or $s = \sqrt{h^2 + (b/2)^2}$

A cone: has one circle base and an apex, has a height and a slant height, can be generated by rotating a right triangle around its height. $s^2 = h^2 + r^2$

A sphere: can be generated by rotating a semicircle around its diameter.

Chapter 6: Areas and volumes of solids

(×) →

King **H**enry's **D**ead **M**other **D**idn't **C**are **M**uch

← (÷)

Km^3 hm^3 dam^3 m^3 dm^3 cm^3 mm^3

1D: ×/÷ 10 each time
Area: ×/÷ 100 each time
Volume: ×/÷ 1000 each time

KL = m^3 HL DAL L = dm^3 DL CL ML = cm^3

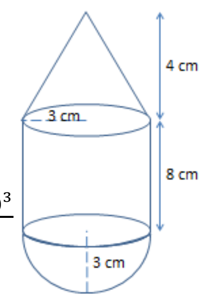
Shape	A_L (unit²)	A_T (unit²)	VOLUME (unit³)
Cube	$4w^2$	$6w^2$	w^3
Prism	$P_b h$	$P_b h + 2A_b$	$A_b h$
Cylinder	$2\pi rh$	$2\pi rh + 2\pi r^2$	$\pi r^2 h$
Pyramid	$\frac{P_b\, s}{2}$	$\frac{P_b\, s}{2} + A_b$	$\frac{A_b\, h}{3}$
Cone	πrs	$\pi rs + \pi r^2$	$\frac{\pi r^2\, h}{3}$
Sphere	$A_{LAT} = A_{TOT} = 4\pi r^2$		$\frac{4\pi r^3}{3}$
Hemisphere	$2\pi r^2$	$3\pi r^2$ If base is included	$\frac{2\pi r^3}{3}$

P_b: Perimeter of base; A_b: area of base; r: radius; s: slant height; h: height

EX: Find the total surface area and volume of this solid

$A_T = A_{Lcone} + A_{Lcylinder} + A_{Lhemisphere}$
$= \pi rs + 2\pi rh + 2\pi r^2$
$= \pi(3)(5) + 2\pi(3)(8) + 2\pi(3)^2$
$= 15\pi + 48\pi + 18\pi$
$= 81\,\pi\ cm^2$ or $254.47\ cm^2$

$V = V_{cone} + V_{cyliner} + V_{hemisphere}$
$= \frac{\pi r^2\, h}{3} + \pi r^2 h + \frac{2\pi r^3}{3}$
$= \frac{\pi(3)^2\,(4)}{3} + \pi(3)^2(8) + \frac{2\pi(3)^3}{3}$
$= 12\pi + 72\pi + 18\pi$
$= 102\,\pi\ cm^3$ or $320.44\ cm^3$



$A = x^2$; $P = 4x$

$A = \frac{bh}{2}$; $P = a+b+c$

$A = bh$; $P = 2(b+h)$

$A = \pi r^2$; $C = \pi d$

$A = \frac{(B+b)h}{2}$

Chapter 7: Similar solids

Similar solids have a ratio k between them (each dimension of one is k times bigger than the other) it is easier to keep k as a fraction instead of a decimal.

1D: ratio of sides $k = \frac{s'}{s} = \frac{h'}{h} = \frac{b'}{b} = \frac{l'}{l} = \frac{r'}{r}$

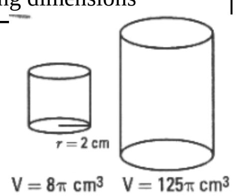
2D: ratio of areas $k^2 = \frac{A'}{A}$

3D: ratio of volumes $k^3 = \frac{V'}{V}$

1. Solve for k first
2. Set up equal ratios
3. Cross multiply to solve for missing dimensions

EX: Find the Area of base of the big cylinder

$k^3 = \frac{V'}{V}$ $k^3 = \frac{125\pi}{8\pi} = \frac{125}{8}$ $k = \sqrt[3]{\frac{125}{8}} = \frac{5}{2}$	$k = \frac{r'}{r}$ $\frac{5}{2} = \frac{r'}{2}$ $r' = 5$	$A_b = \pi r^2$ $= \pi(5)^2$ $= 25\,\pi\ cm^2$



Chapter 8: Probabilities

Basic counting principle:

	With repetition (with replacement)	Without repetition (without replacement)	
Permutations (with order)	n^r *EX: roll a die, twice: (6)(6)=36*	• All n items: $n!$ *EX: arrange 5 books: 5! = 120*	• r out of n items: $nPr = P_r^n = \frac{n!}{(n-r)!}$ **Or** n (n - 1) (n - 2) … r times *EX: Give 3 awards to 7 students: (7)(6)(5) = 7P3 = 210*
Combination (without order)	$\frac{(n+r-1)!}{(n-1)!\,r!}$ *EX: give 5 stickers in a class of 10: 14! / (9!5!) =2002*	$nCr = C_r^n = \frac{n!}{(n-r)!\,r!}$ *EX: Choose 6 horses from 8 in total: 8C6 =28*	

n is the total available items/choices
r is the number of items/choices needed

- Probability of an event: $P(A) = \frac{\#\ of\ desired\ outcomes}{total\ \#\ of\ outcomes}$

Geometric probability:

- 1D: $P(target) = \frac{target\ length}{total\ length}$
- 2D: $P(target) = \frac{target\ area}{total\ area}$
- 3D: $P(target) = \frac{target\ volume}{total\ volume}$

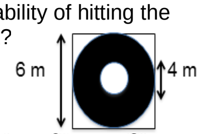
A intersection B: $(A \cap B)$ is the event when A **and** B both occur.

A union B: $(A \cup B)$ is the event when A **or** B occur.

Compliment of A ($\bar{A}$ or A') is the event when anything except A occurs.

P(A and B) = P(A) x P(B)
P(A or B) = P(A) + P(B)

EX: What is the probability of hitting the black?



$P(B) = \frac{A_{Bcircle} - A_{Scircle}}{A_{square}}$
$= \frac{\pi R^2 - \pi r^2}{s^2}$
$= \frac{\pi(3)^2 - \pi(2)^2}{6^2}$
$= \frac{9\pi - 4\pi}{36}$
$= \frac{5\pi}{36} = 0.436$

Chapter 9: Statistics

Type of survey:

CENSUS: whole population; **POLL**: small sample; **STUDY**: Experts in topic

Type of variable/data: 1-**QUALITATIVE** (quality/non numerical); or 2-**QUANTITATIVE**(numerical): **Discrete** (Integers); or **Continuous** (Real #s)

Sampling Methods: 1-**Random**; 2-**Systematic**;
3- **Cluster**: randomly choosing some clusters and surveying them whole
4- **Stratified**: all proportions (%) of the populations are represented FAIRLY

EX: from a school of 1200 students a sample of 180 is chosen. How many girls from grade 8 should be in the sample?

	Girls	Boys
Grade 7	360	345
Grade 8	240	255

$\frac{240}{1200} = \frac{x}{180}$ therefore x = 36

Measures of central tendency:

Mode(Mo): value with the highest frequency

Median(Md): value at the center of an ordered list

Mean($\bar{x}$): average of all values of data

Height	Freq.	Midpoint	Total height
[100,110[	8	105	8(105)=840
[110,120[	2	115	2(115)=230
[120,130[	7	125	7(125)=875
[130,140[	2	135	2(135)=270
Total	19		2215

EX: **Mo** = [100,110[;

Md = the tenth class: [110,120[; $\bar{x} = \frac{2215}{19} = 116.58$

Measures of position:

Quartiles: Q1, Q2, Q3 divide the ordered list of data into 4 groups containing the same number of data in each. The **Q2** is the median, we find it first.

Measures of dispersion:

Range: $R = X_{max} - X_{min}$

Interquartile range: I = Q3 – Q1

Tables and diagrams:

1. Bar graph 2. Pie chart 3. Broken line graph 4. Histograms
5. **Box and Whiskers plot.** EX: