

Practice Exam #1

C1

BIRD FEEDERS

A company that makes bird feeders sells different kits online. Each kit comes in a cardboard box and contains three feeders: a green one, a red one and a yellow one. The company uses a courier service to deliver the boxes to customers' homes.

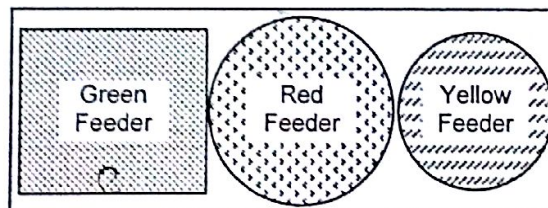
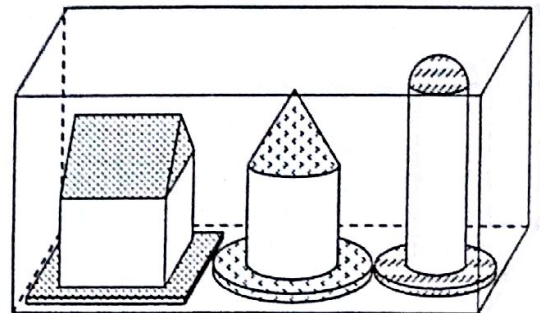
Note: All costs include taxes (no need to calculate the taxes).
Figures are not drawn to scale.

Your task: you must determine the cost of delivering one of the kits.

- Each of the three feeders is made up of a tray, a reservoir and a lid.

CARDBOARD BOX

- The cardboard box used to package the feeders will be in the shape of a right prism with a rectangular base.
- The trays are centered along the bottom of the box.
- The lengths of the edges (in cm) of the box must be integers.
- The lengths of the edges of the box must be greater than the minimum lengths required for the feeders to fit, but by a maximum of 1 cm.
- The figure below shows how the feeder trays are laid out in the box (top view). The longest sides of the tray of the green feeder are parallel to the longest sides of the box.



THE COST OF DELIVERING A KIT

- The courier service determines the cost of delivering a box according to its volume.

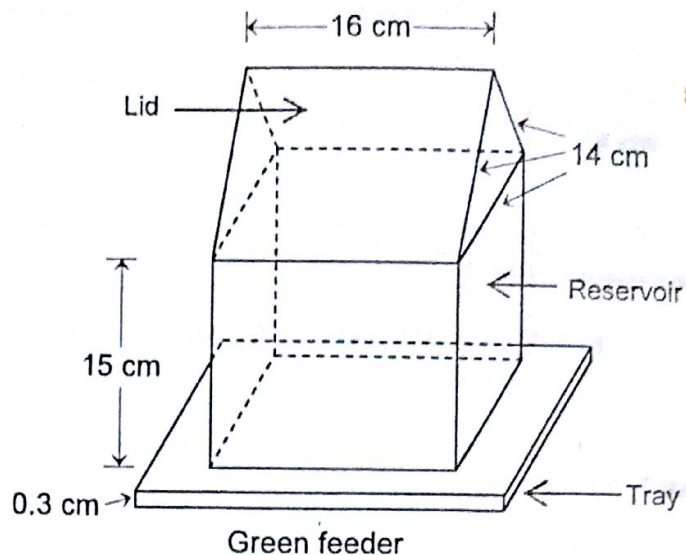
The table of values below lists different boxes with a volume of more than 50 dm^3 , and their respective delivery cost.

COST OF DELIVERING A BOX ACCORDING TO ITS VOLUME

Volume (dm^3)	Cost (\$)
55	6.00
57	6.80
68	11.20
75	14.00

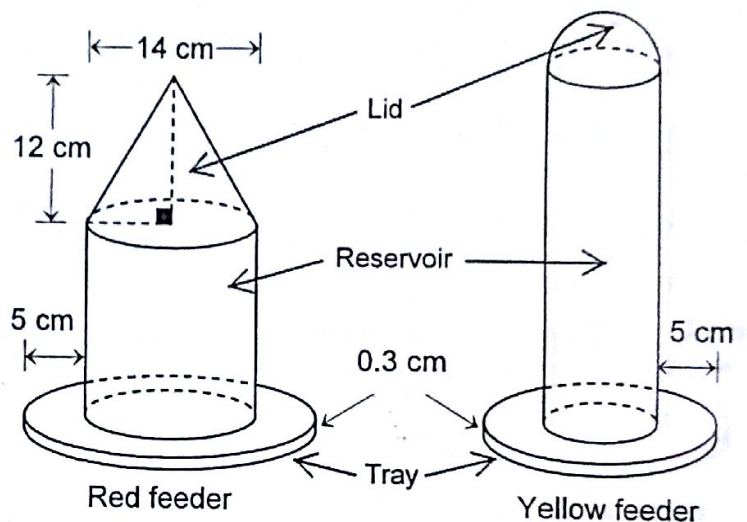
THE GREEN FEEDER

- The reservoir is a rectangular prism. Its height is 15 cm. The base of the reservoir measures 16 cm by 14 cm.
- The tray is also a rectangular prism. The tray is 0.3 cm thick.
- The base of the reservoir is similar to the bases of the tray. The area of the base of the tray is equal to 2.25 times the area of the base of the reservoir.
- The lid is a triangular prism. The bases of this prism are equilateral triangles with side lengths of 14 cm.



THE RED FEEDER AND THE YELLOW FEEDER

- The reservoirs of these feeders are cylinders. The capacity of both reservoirs must be the same. Also, the capacity must be at least 1.9 litres and at most 2.5 litres.
- The capacities of the cylindrical reservoirs made by this company are in multiples of 100 mL.
- The trays of both feeders are also cylinders with a height of 0.3 cm. The radius of each cylindrical tray is 5 cm greater than the radius of the base of the reservoir of the feeder of the same colour.
- The lid of the red feeder is a cone. Its height is 12 cm. The bases of the reservoir and the lid of the red feeder have a diameter of 14 cm.
- The lid of the yellow feeder is a half sphere. Its radius is 5 cm and is equal to the radius of the reservoir.



THE HEIGHT OF THE YELLOW FEEDER

- The height of the yellow feeder must be greater than the height of each of the other two feeders in the kit.

You must determine the cost of delivering one of the kits.

Indicate this cost at the bottom of page 11 in the *Answer Booklet*.

①

Show all your work

To find: ① $l = \text{length of } G + d \text{ of } R + d \text{ of } Y + 1 \text{ cm}$

$w = \text{width of } G \text{ OR } d \text{ of } R$

$h = \text{height of } Y + 5 \text{ cm} + 0.3 \text{ cm}$

② Volume of Box = $l \times w \times h$

③ Cost of delivery: $y = ax + b$.

COST: $y = ax + b$

$$a = \frac{\Delta y}{\Delta x} = \frac{6.80 - 6.00}{57 - 55} = \frac{0.80}{2} = 0.4$$

$$y = 0.4x + b$$

$$6.00 = 0.4(55) + b$$

$$6.00 = 22 + b$$

$$-22 \quad -22$$

$$b = -16$$

$$\therefore y = 0.4x - 16$$

GREEN FEEDER:

$$\frac{A_{\text{Tray}}}{A_{\text{base}}} = 2.25 = K^2 \Rightarrow K = \sqrt{2.25} = 1.5$$

$$\frac{l_{\text{Tray}}}{l_{\text{base}}} = \frac{x}{16} = 1.5 \Rightarrow l = 24$$

$$\frac{w}{14} = 1.5 \Rightarrow w = 21$$

$$\left[\begin{array}{l} \text{height of side } c^2 = a^2 + b^2 \\ 14^2 = 7^2 + h^2 \\ 147 = h^2 \\ h = 12.1243 \end{array} \right. \quad \left. \begin{array}{l} \text{height of } G = 12.1243 + 15 + 0.3 \\ = 27.42 \text{ cm} \end{array} \right]$$

RED FEEDER:

$$\text{diameter} = 14 + 10 \Rightarrow d = 24 \text{ cm}$$

$$\left[\begin{array}{l} \text{height of cyl. reservoir if } V = 2.0 \text{ L} = 2000 \text{ mL} = 2000 \text{ cm}^3 \\ V = \pi r^2 h \\ 2000 = \pi (7^2) h \\ h = \frac{2000}{53.94} \\ = 12.99 \end{array} \right. \quad \left. \begin{array}{l} \text{height of } R \text{ feeder} = 12.99 + 12 + 0.3 \\ = 25.29 \end{array} \right]$$

Show all your work

YELLOW FEEDER:

$$\text{diameter} = 10 + 10 \Rightarrow \boxed{d = 20 \text{ cm}}$$

$$\text{height of Y if } V = 2.0 \text{ L} = 2000 \text{ ml} = 2000 \text{ cm}^3$$

h of cyl:

$$V = \pi r^2 h$$

$$2000 = \pi (5^2) h$$

$$h = \frac{2000}{25\pi}$$

$$= \frac{2000}{78.54}$$

$$= 25.46$$

$$h \text{ of Y} = 25.46 + 5 + 0.3$$

$$= 30.76 \text{ cm}$$

$$\text{Check : ① Volume R} = \text{Volume of Y} = 2.0 \text{ L}$$

$$\text{② } 1.9 \text{ L} < V < 2.5 \text{ L}$$

$$1.9 \text{ L} < 2.0 < 2.5 \text{ L}$$

$$\text{③ } h \text{ of Y} > h \text{ of G} > h \text{ of R}$$

$$30.76 > 27.42 > 25.29$$

Volume of BOX:

$$l = l \text{ of G} + d \text{ of R} + d \text{ of Y} + 1 \text{ cm}$$

$$= 24 + 24 + 20 + 1 \Rightarrow \underline{l = 69 \text{ cm}}$$

$$w = (w \text{ of G or } d \text{ of R}) + 1 \text{ cm}$$

$$(21 \text{ or } 24) + 1 \Rightarrow \underline{w = 25 \text{ cm}}$$

$$h = h \text{ of Y} = 31$$

$$h = 31$$

$$\text{Volume} = l \times w \times h = 69 \times 25 \times 31 = 53475 \text{ cm}^3$$

$$= 53.475 \text{ dm}^3$$

COST:

$$Y = 0.4(53.475) - 16 = \$5.39$$