Practice Exam #3

The Full Moon Circus Expansion

The owner of a traveling circus would like to increase his profits. To do this, he needs a larger "Big Top Tent" to accommodate more spectators.

You must determine the minimum price of a ticket to see the circus while taking the following into consideration:

Conditions to be respected when determining the ticket price:

- **The mean number of people attending each performance under the new "Big Top Tent" will be 90% of the total seating capacity**
- **There are 450 circus performances per year**
- **The profit generated from ticket sales must be at least 15% of the total expenses**
- **The ticket price must be a whole number**

The Number of Seats under the new "Big Top Tent"

The following list shows the mean attendance for each circus performance over the last 12 years.

890, 900, 870, 960, 945, 995, 995, 965, 1000, 1000, 1000, 1000

The new "Big Top Tent" will have 25% more seats than the mean attendance over the last 12 years.

Weight of the new "Big Top Tent" (includes steel frame and canvas top)

- **The Steel Frame**

The 11 weights below represent the different weights of the steel frame of the "Big Top Tent" that can be constructed. The weights are measured in kilograms.

1825 2600 1915 2450 2625 1875 2200 2715 1750 2200 1875

The weight of the new "Big Top Tent" steel frame is equivalent to the 3rd quartile.

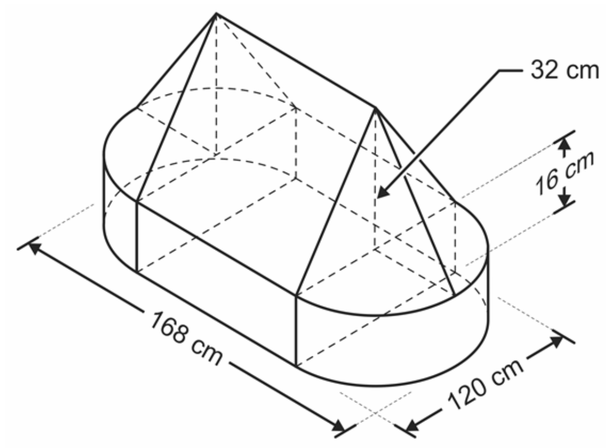
- **The Canvas Top**

The canvas top covers the exterior of the new "Big Top Tent" completely (the roof and the sides, but not the floor). The table below shows the relation between the number of seats under the new "Big Top Tent" and its respective volume, in m^3.

Number of seats	Volume of the new "Big Top Tent" in m^3
600	3993.75
850	5493.75

In order to help the circus owner determine the amount of canvas needed, the manufacturing company has provided a scale model that is similar to the new "Big Top Tent" (shown below).

- The volume of the scale model is 0.486 m^3
- The weight of the canvas top is a function of its surface area
- The weight of the canvas top is 0.5 kg/m^2



<u>The Cost of the new "Big Top Tent"</u>

The cost of the new "Big Top Tent" depends on its total weight (steel frame and canvas top) and is determined by the following linear function:

$$y = 325x + 6675$$

where x is the total weight, in kg, rounded to the nearest whole number
y is the cost in dollars

<u>Transportation Costs</u>

In order for the circus to travel, a transport truck is required to move the "Big Top Tent" from city to city. The new "Big Top Tent" is larger than the old one so a new truck is needed to transport it. The cost of a transport truck depends on the total weight of the new "Big Top Tent":

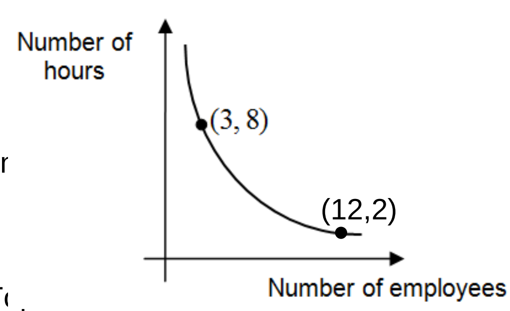
	Total weight of the new "Big Top Tent"	**Cost of a new Transport Truck**
1st Model	Weight ≤ 2500 kg	$ 95 000
2nd Model	2500 < Weight ≤ 4000 kg	$ 120 000
3rd Model	Weight > 4000 kg	$ 175 000

<u>Advertisement Costs</u>

The cost to advertise the circus for 1 year is $ 16 545 000.

<u>Labor Costs</u>

The graph shows the inverse relation between the number of er set up the new “Big Top Tent”.



It is determined that it will take 4 hours to set up the new “Big T(.

Each employee hired to help set up the new “Big Top Tent” will receive an annual salary of $35 000.

Determine the minimum price of a ticket to see the circus while taking the following into consideration:

1) The Number of Seats under the new 'Big Top Tent'

$$\text{mean} = \frac{890+900+870+960+945+995+995+965+(4x1000)}{12} = \frac{11\ 520}{12} = 960$$

The number of seats is 25% more than the mean: 960 × 1.25 = 1200 seats

2) The Weight of the new 'Big Top Tent'

2a) The Weight of the Steel Frame

Place the given weights in increasing order:

1750, 1825, 1875, 1875, 1915, 2200, 2200, 2450, 2600, 2625, 2715

Q_1= 1875 Q_2= 2200 Q_3= 2600

Given that the new 'Big Top Tent' is equivalent to the 3rd quartile, the weight of the steel frame is therefore 2600 kg.

2 b) The Weight of the Canvas Top

- **The volume of the new 'Big Top Tent' using the table of values**

$$y = ax + b$$

$$a = \frac{5493.75 - 3993.75}{850 - 600}$$

$$a = \frac{1500}{250}$$

$$a = 6$$

$$y = 6x + b$$

$$5493.75 = 6(850) + b$$

$$393.75 = b$$

The rule is: $y = 6x + 393.75$

Calculate the volume given that it seats 1200 people → x= 1200

$$y = 6(1200) + 393.75$$

$$y = 7593.75 \text{ m}^3$$

The volume of the new 'Big Top Tent' = 7593.75 m^3

- **The similarity ratio (K) between the new 'Big Top Tent' and the scale model**

$$k^3 = \frac{7593.75}{0.486} = 15\,625$$
$$k = \sqrt[3]{15\,625} = 25$$

The similarity ratio between the sides is k = 25

The similarity ratio between the areas is $k^2 = 25^2 = 625$

- **The surface area of the canvas top on the scale model**

Total Surface $\text{Area}_{\text{Model}}$ =
Lateral $\text{Area}_{\text{Cylinder}}$ + Lateral $\text{Area}_{\text{Cone}}$ + Lateral $\text{Area}_{\text{Large Rectangle}}$ +
Lateral $\text{Area}_{\text{Small Rectangle}}$

$$\begin{aligned}\text{Lateral Area}_{\text{Cylinder}} &= 2\pi r h \\ &= 2\pi \bullet 60 \bullet 16 \\ &= 1920\pi \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Lateral Area}_{\text{Cone}} &= \pi r \text{SL} \\ &= \pi \bullet 60 \bullet 68 \\ &= 4080\pi \text{ cm}^2\end{aligned}$$

Slant Height

$$\begin{aligned}\text{SL}^2 &= 60^2 + 32^2 \\ \text{SL}^2 &= 4624^2 \\ \sqrt{\text{SL}^2} &= \sqrt{4624^2} \\ \text{SL} &= 68 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Lateral Area}_{\text{Large Rectangle}} &= L \bullet \ell \bullet 2 \\ &= 68 \bullet 48 \bullet 2 \\ &= 6528 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Lateral Area}_{\text{Small Rectangle}} &= L \bullet \ell \bullet 2 \\ &= 48 \bullet 16 \bullet 2 \\ &= 1536 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total Surface Area}_{\text{Model}} &= 1920\pi + 4080\pi + 6528 + 1536 \\ &= 26\,913.56 \text{ cm}^2\end{aligned}$$

- **Surface area of the canvas for the new ‘Big Top Tent’, using the similarity ratio between areas**

26 913.56 × 625 = 16 820 975 cm^2 = 1682.0975 m^2 = 1682.1 m^2

- **The weight of the new ‘Big Top Tent’ canvas top**

$$1682.1\text{m}^2 \times \frac{0.5 \text{ kg}}{\text{m}^2} = 841.050 \text{ kg}$$

The total weight of the new ‘Big Top Tent’

Total weight = weight of the steel frame + weight of canvas top

Total weight = 2600 kg + 841.05 kg = 3441.05 kg

Total weight = 3441 kg (rounded to the nearest whole number)

3) The total expenses of the new ‘Big Top Tent’

3a) The cost of the new ‘Big Top Tent’

Total weight of the ‘Big Top Tent’ is 3441 Kg → x= 3441

$y = 325x + 6675$
$y = 325\ (3441) + 6675$
y = 1 125 000

Cost = $1 125 000

3b) Transportation Costs

Total weight of the ‘Big Top Tent’ is 3441 Kg → 2nd model, between 2500 kg and 4000 kg

Cost of the transport truck = $120 000

3c) Advertisement Costs

Advertisement cost is $16 545 000 (given)

3d) Labor costs

$$y = \frac{k}{x}$$

$$8 = \frac{k}{3}$$

$$k = 24$$

$$y = \frac{24}{x}$$

$$4 = \frac{24}{x}$$

$$4x = 24$$

$$x = 6 \text{ employees}$$

Cost of the 6 employees is 6 x $35 000 = $210 000

Total expenses of the New ‘Big Top Tent’

Total expenses = (steel frame + canvas top) + transportation costs + advertising costs + additional employee’s salary

Total expenses = 1 125 000 + 120 000 + 16 545 000 + 210 000

Total expenses = $18 000 000

4) Price of a ticket

- **The mean number of people attending each performance will be 90% of the total seating capacity**

  Number of Seats per year: 1200

  90% of the mean number of seats: 1200 × 0.9 = 1080 seats

- **There are 450 circus performances per year**

  450 performances a year: $1080 \times 450 = 486\,000$ seats per year

- **The profit generated must be ≥ 15% of total expenses**

  Total Expenses: $18 000 000

  Total Profit: $18 000 000 × 0.15 = $2 700 000

  The Revenue from ticket sales

  $18 000 000 + $2 700 000 = 20 700 000

- **The ticket price**

  Let x = price of a ticket

  (Number of seats per year)(price of a ticket) ≥ Total sales

  $486\,000\,x \geq 20\,700\,000$

  $\underline{x \geq 42.59}$

  The price of a ticket must be a whole number: $x = \$43$

Answer:

The minimum price of a ticket to see the circus will be $43.