8.1 Basic counting principle

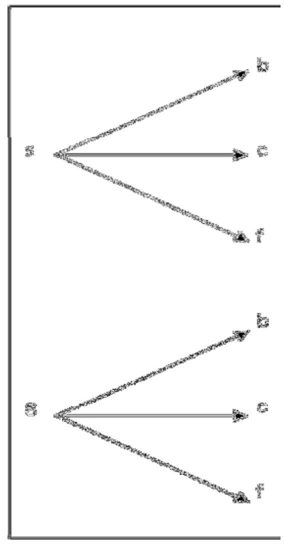
EX 1: If the menu at a restaurant has the following choices:

Appetizer: soup or green salad

Main course: beef, chicken or fish

Dessert: pie or ice cream

How many possible outcomes (combinations of meals) are there?



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- **Appetizer**: soup or green salad
- **Main course**: beef, chicken or fish
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How many possible outcomes (combinations of meals) are there?

2 x 3 x 2 = 12 possible outcomes

Appetizer Main Dessert

Basic Counting Principle

If there are m ways to do one thing,
and n ways to do another,
then there are m x n ways of doing both.

EX 2: How many outfits can be worn with 4 different shirts, 3 pants and 3 pairs of shoes.

Ex 3: How many outcomes are there when

a) Rolling 1 die

d) Flipping a coin

b) Rolling 2 dice

e) Flipping a coin 3 x

c) Rolling 3 dice

f) Flipping a coin 3 x and rolling a dice 2 x

Ex 4: How many possible Quebec license plates start with 3 numbers followed by 3 letters?

How about in Ontario?

How about if no repetition is allowed?

Practice:
Page 229 # 1-9

8.2 –A- Arrangements, Permutations

A Permutation is an ordered arrangement where ALL or SOME of the items in a set are used.

EX 1. How many ways can 8 athletes receive gold, silver and bronze medals?

Ex 2 How many 4 letter sequences can be made with the vowels a,e,i,o,u & y without repeating?

Ex 3: How many different ways can you arrange 6 books on the shelf?
(order matters and there is no repetition of a book)

There is a notation for writing this in short:
6! We read it 6 factorial.
On the calculator it is n!.
n! = n x (n-1) x (n-2) x .....x 3 x 2 x 1.
Note that 0! = 1

Evaluate these Factorials

$4! =$ ________ $\qquad \frac{8!}{3!} =$ ________

$5! =$ ________ $\qquad \frac{11!}{7!} =$ ________

$9! =$ ________ $\qquad \frac{10!}{2!6!} =$ ________

Ex 4: If out of the 6 books, **4 are French** and **2 are English**.
How many ways can we arrange them if:

a) We want to keep the same languages together?

Ex 4: If out of the 6 books, **4 are French** and **2 are English**.
How many ways can we arrange them if:

b) We want just French together?

Ex 4: If out of the 6 books, **4 are French** and **2 are English**.
How many ways can we arrange them if:

c) We want just English together?

Ex 5: A die is thrown 2 times and the results are recorded.
(order matters and repetition is allowed)

Practice:
Page 230 # 1-4

Why is 0! = 1

8.2 –B- Combinations

A Combination is an arrangement of SOME items chosen. The order does NOT matter.

Case 1: No repetition/replacement

Ex 1: Sheila has 4 shirts (pink, blue, yellow, green), she wants to choose 2 for a trip.

We can use a formula for this :

$$nCr = C_r^n = \frac{n!}{(n-r)!\,r!}$$

We read this: n choose r

Where:

n is the number of total choices available

r is the # steps/items to be chosen

Ex 2: A store has 6 employees, but only 3 need to be on duty at any time.

Ex 3: A committee of 3 people must be formed from a club of 5 members. How many different committees are possible?

Ex 4: How many 6-number combinations are there in the lottery game 6/49?

Case 2: with repetition/replacement

Again we can use a formula for this:

$$\frac{(n+r-1)!}{(n-1)!\,r!}$$

Ex 1: How many combinations with repetition can be made from 10 objects taking 4 at a time?

Ex 2: Two prizes are awarded in a class of 20 students. A student can win both prizes. How many different pairs of winners are possible if the order in which the prizes are awarded is not considered?

Practice:
page 232 # 5-10

Permutation or Combination?

A) One chooses 3 different toppings on a tofu burger from a choice of 15 toppings.

Combination

Permutation

Permutation or Combination?

B) Arrange all 6 shirts in your closet. Order is important.

Combination

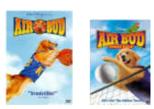
Permutation

Permutation or Combination?

C) Take 2 of your <u>favourite</u> movies from a collection of 15 dvds to a friend's for a slumber party.

Combination

Permutation



Permutation or Combination?

D) 3 cards from a deck are dealt, order is important.

Combination

Permutation

Permutation or Combination?

E) A team of 6 horses from a batch of 8 horses are chosen.

Combination

Permutation

You have 5 books on the shelf in how many ways can you…

a) Order them?

b) Read only 5 in order with possible repetition?

c) Pick only 3 in order with possible repetition?

You have 5 books on the shelf in how many ways can you…

d) Pick 3 in order without repetition?

e) Pick any 3 at a time without replacement?

f) Pick any 3 at a time with replacement?

Practice:
Worksheet “Extra Practice”

8.3 –A- Probability of events

Recall:

- Random experiment is one that depends entirely on chance.
- Sample space Ω (omega) is the set of all possible outcomes
- An Event is a subset of the sample space.
  - Simple event: contains a single outcome from the sample space.
  - Compound event: contains a series of simple events.

Determine the sample space for a random experiment

a) Your favorite subject in school.

$\Omega=$

b) Flipping a coin 3 times in a row

$\Omega=$

c) For rolling a die once

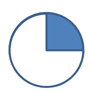
$\Omega=$

An event of "rolling a # greater than 2" : corresponds to {3,4,5,6}
A simple event is "rolling a 1" because it corresponds to {1}

The probability can be a fraction, a decimal (between 0 and 1), or a percentage. (0 being impossible, and 1 being certain)

Theoretical Prob = $\dfrac{\text{\# of desired outcomes}}{\text{total \# of outcomes}}$

Ex: P(randomly choosing a point in the dark sector) = $\frac{1}{4}$



Experimental Prob = $\dfrac{\text{\# of desired outcomes observed}}{\text{\# of trial runs}}$

Ex: The experimental probability that a hockey team will win the Stanley cup is based on its performance in previous games.

- The more times a random experiment is repeated, the closer the experimental probability gets to the theoretical probability.

What is the probability of picking the correct 6 numbers out of the 49 to win the Lotto 649?
(order doesn't matter, and with no repetition)

Prob = $\dfrac{\text{\# of desired outcomes}}{\text{total \# of outcomes}}$

8 students are auditioning for a part in the school musical.
Adam, Bob, Carl, Dan, Ed, Frank, George, and Howard.
If only 6 will be chosen, what is the probability that it will be: Bob, Carl, Dan, Ed, Frank and Howard?

Prob = $\dfrac{\text{\# of desired outcomes}}{\text{total \# of outcomes}}$

The AND of Probability: Think MULTIPLY

When 2 independent EVENTS happen in a row, the probability of event 1 AND event 2 occurring is the multiplication of the probability of each individual event.
P(A and B) = P(A,B)= P(A).P(B)

a) Drawing a Queen & Rolling a 6.

b) Drawing a Spade & Rolling an even #.

The OR of Probability: Think ADD

When two independent events happen and one event OR the other is considered a success, the probability of either occurring is the ADDITION of the probabilities of each individual event.

P(A or B) = P(A) + P(B)

Find the probability

a) Rolling a 1, 2 or 4

b) Drawing a King or Jack

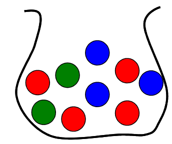
Draw a probability tree for picking 2 marbles from a bag, in order and **without replacement**. The bag contains 4 RED, 3 BLUE and 2 GREEN.

Red $\frac{4}{9}$
- Red $\frac{3}{8} = \left(\frac{4}{9}\right)\left(\frac{3}{8}\right) = \frac{12}{72}$
- Blue $\frac{3}{8} = \frac{12}{72}$
- Green $\frac{2}{8} = \frac{8}{72}$

Blue $\frac{3}{9}$
- Red $\frac{4}{8} = \frac{12}{72}$
- Blue $\frac{2}{8} = \frac{6}{72}$
- Green $\frac{2}{8} = \frac{6}{72}$

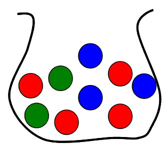
Green $\frac{2}{9}$
- Red $\frac{4}{8} = \frac{8}{72}$
- Blue $\frac{3}{8} = \frac{6}{72}$
- Green $\frac{1}{8} = \frac{2}{72}$

The sum of all probabilities =1



Determine the probability of picking

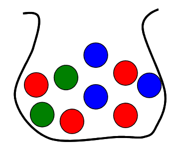
a) a RED or a GREEN marble



b) 2 marbles with at least 1 BLUE marble

Determine the probability of picking

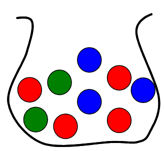
c) 2 RED marbles



d) 2 of the same colour twice

Determine the probability of picking

e) 2 Blue marbles one after the other and with replacement



f) a RED and a GREEN marble with replacement

Sophie has to take two exams. She estimates that she has a $\frac{1}{3}$ chance of passing the first exam and a $\frac{3}{5}$ chance of passing the second exam.

What is the probability of her passing only one exam?

A) $\frac{4}{15}$

B) $\frac{7}{15}$

C) $\frac{8}{15}$

D) $\frac{11}{15}$

Practice: Page 238 # 7-12

8.3 –B- Geometric Probability

In any random experiment there are two types of random variables:

Discrete Random Variable:

If it cannot take on all the possible values of an interval of real numbers.

Ex.: We roll two dice and observe the outcome.

We are interested in the **sum of the two outcomes.**

Continuous Random Variable:

If it can take on all the possible values of an interval of real numbers.

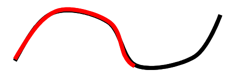
Ex.: We randomly choose a checkout in a grocery store.

We are interested in the **waiting time** for the people in line.

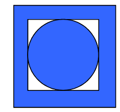
Geometric Probability

There are 3 types of geometric probabilities, one for each of the commonly used dimensions of space; length, area and volume.

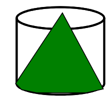
1D P(Target) = $\frac{\text{Target length}}{\text{Total length}}$



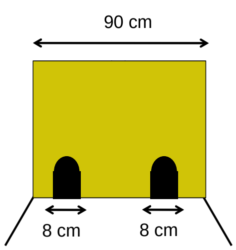
2D P(Target) = $\frac{\text{Target area}}{\text{Total area}}$



3D P(Target) = $\frac{\text{Target volume}}{\text{Total volume}}$



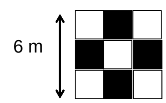
Ex 1: What is the probability that the blind mouse will escape into a hole?



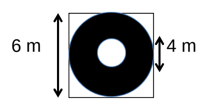
P(Target) = $\frac{\text{Target length}}{\text{Total length}}$

Ex 2: Which black target is a skydiver most likely to land on?

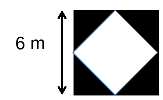
Target A



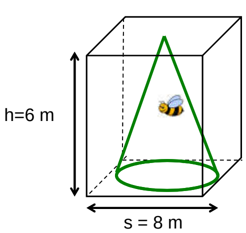
Target B



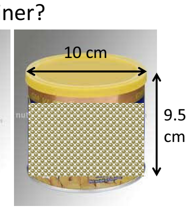
Target C



Ex 3: What is the probability the bee is in the laser cone?



Ex 4: The NUT HOUSE factory has two types of containers, a square base prism and a cylinder. Each hour they package 20 of the prism and 25 of the cylinder. Between 2 pm and 3 pm, they had some problem with their machine and lost one of their bolts in one of the containers. What is the probability that it fell in a cylinder container?



8.3 –C- Operations between events

Ex 1: Roll a fair die once.

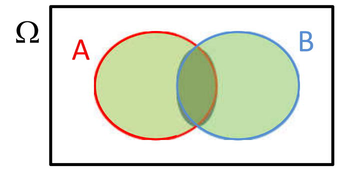
Ω= {1,2,3,4,5,6}

Consider:

Event A: rolling an even number

Event B: rolling a number less than 4

The 2 events can be represented by the Venn Diagram here



1) A **Intersection** B: $A \cap B$ is the event when A and B both occur.

2) A **Union** B: $A \cup B$ is the event when A or B occur.

3) Complement of A: $\bar{A}$ or A' (read A bar or A prime, and means contrary of A): is the event when anything except A could occur.

So in Ex 1: Roll a fair die once.

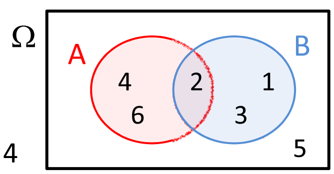
Ω= {1,2,3,4,5,6}

Event A: rolling even #

A = {2,4,6}

Event B: rolling a # less than 4

B = {1,2,3}



$\bar{A}$ =

$\bar{B}$ =

$A \cap B$ =

$A \cup B$ =

$A \cap \bar{B}$ =

$\bar{A} \cap B$ =

Practice:

page 242 # 21-23